

Shikshan Mandal Karad's
Mahila Mahavidyalaya Karad
B.Sc. (Part-I) (Preliminary -I) Examination
DSC-A5: MATHEMATICS-I(CBCS)
Calculus

Day and Date: Thursday and 06/01/2023

Time: 1:00 PM to 3:00 PM

Total Marks: 40

Instructions: 1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q1) Choose the most correct alternative

[8]

- a) The infinite series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \dots$ is expansion of _____

 - i) $\cos x$
 - ii) $\sin x$
 - iii) e^x
 - iv) a^x

b) If $f(x) = e^x, x \in [0,1]$ then the value of c of L.M.V.T. IS _____

 - i) $e - 1$
 - ii) $\frac{1}{e-1}$
 - iii) $e(e - 1)$
 - iv) $\log(e - 1)$

c) Which of the following is a mean value theorem

 - i) Euler's Theorem
 - ii) Leibnitz's Theorem
 - iii) Taylor's Theorem
 - iv) Rolle's Theorem

d) If $f(x) = |x|$ then $f(x)$ is at $x = 0$.

 - i) Continuous
 - ii) Derivable
 - iii) Discontinuous
 - iv) None

e) If neither $\lim_{n \rightarrow c^-} f(x)$ nor $\lim_{n \rightarrow c^+} f(x)$ exist, then the function is _____.

 - i) Discontinuous of first kind
 - ii) Discontinuous of second kind
 - iii) Continuous
 - iv) Differentiable

f) Left hand limit of $f(x) = 2x^2 - 1$ for $0 \leq x \leq 2$
 $= 4x + 1$ for $2 \leq x \leq 4$

as $x \rightarrow 2$ is _____

- i) 7
iii) 6

- ii) 9
iv) -1

g) Every differentiable function is _____.

- i) Continuous
iii) Removable discontinuous

- ii) Discontinuous
iv) None of the above.

h) If $y = e^{3x}$ then y_n is _____.

- i) e^{3nx}
iii) 3^n

- ii) $3^n e^{3x}$
iv) e^{3x}

Q2) Attempt any two of the following.

[16]

a) State and prove Leibnitz's theorem.

b) State and prove Lagrange's mean value theorem.

c) If $y = (x^2 - 1)^n$ then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n-1)y_n = 0$.

Q3) Attempt any four of the following.

[16]

a) Evaluate $\lim_{n \rightarrow \infty} \sqrt{x} [\sqrt{x+1} - \sqrt{x}]$

b) Verify Cauchy's M.V.T. for $2x^3$ and x^6 in $[a, b]$ and find c.

c) Find n^{th} derivative of $y = \sin^2 x$.

d) Expand $\log(1+x)$ I power of x.

e) Evaluate $\lim_{n \rightarrow 0} \frac{\log(5+x) - \log(5-x)}{x}$.

f) Verify Rolle's theorem $f(x) = x^3 - 4x$ on $[-2, 2]$.

Shikshan Mandal Karad's
Mahila Mahavidyalaya Karad
 B.Sc. (Part-I) (Preliminary-I) Examination
 DSC-A6: MATHEMATICS-II (CBCS)
 Differential Equations

Day and Date: Thursday and 07/01/2023

Time: 1:00 PM to 3:00 PM

Total Marks: 40

Instructions: 1) All questions are compulsory.
 2) Figures to the right indicate full marks.

Q1) Choose the most correct alternative

[8]

- a) Find out order and degree $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$
- | | |
|--------------|--------------|
| i) 2, 3 | ii) 2, 1 |
| iii) 1, 3 | iv) 2, 2 |

- b) The integrating factor of the equation $\frac{dx}{dy} + py = Q$ is ____.
- | | |
|-----------------------|----------------------|
| i) $e^{\int p dy}$ | ii) $e^{\int p dx}$ |
| iii) $e^{-\int p dx}$ | iv) $e^{-\int p dy}$ |

- c) The general solution of the differential equation $\sqrt{y + px} = p$ is ____.
- | | |
|---------------------|---------------------|
| i) $y = cx^2 + c$ | ii) $y = -cx + c^2$ |
| iii) $y = cx + c^2$ | iv) $y = cx + c$ |

- d) The value of $\frac{1}{D^2+2} \sin x$ is ____.
- | | |
|-------------------------|---------------|
| i) $-\cos x$ | ii) $-\sin x$ |
| iii) $\frac{\sin x}{3}$ | iv) $\sin x$ |

- e) The solution of the equation $p^2 - 9p + 18 = 0$ is ____.
- | | |
|------------------------------------|------------------------------------|
| i) $(y - 6x - c)(y - 3x - c) = 0$ | ii) $(y + 6x - c)(y - 3x - c) = 0$ |
| iii) $(y - 6x - c)(y - x - c) = 0$ | iv) $(y - 9x - c)(y - 2x - c) = 0$ |

- f) The orthogonal trajectory of $xy = c^2$ is ____.
- i) $x^2 + y^2 = a^2$
 - ii) $x^2 = by$
 - iii) $x^2 - y^2 = a^2$
 - iv) $y = bx$
- g) Differential equation $(x^2 - 3xy^2)dx + (1 + 6y^2 - 3x^2y)dy = 0$ is ____.
- i) Exact
 - ii) Variable Separable
 - iii) Homogeneous
 - iv) Linear
- h) $\frac{1}{D^2 + a^2} \cos ax = \text{_____}$.
- i) $-\frac{x}{2a} \sin ax$
 - ii) $\frac{x}{2a} \sin ax$
 - iii) $-\frac{x}{2a} \cos ax$
 - iv) $\frac{x}{2a} \cos ax$

Q2) Attempt any two of the following.

[16]

- a) Define exact differential equation. Prove necessary and sufficient condition for the differential $Mdx + Ndy = 0$ be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
- b) Prove that $\frac{1}{f(D)} e^{ax} = \frac{x^2}{2!} \frac{1}{\Psi(a)} e^{ax}$ where $f(D) = (D-a)^2 \Psi(D)$ and $\Psi(a) \neq 0$.
- c) Solve the differential equation $(D^4 - 1)y = x \sin x$.

Q3) Attempt any four of the following.

[16]

- a) Find out the orthogonal trajectories of $r = a\theta$.
- b) Solve $(x^2 + y^2 + e^x)dx + 2xy dy = 0$
- c) Define clairaut's equation and explain the method of solving it.
- d) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
- e) Solve $(x+2y^3)\frac{dy}{dx} = y$
- f) Solve $(D^3 - 3D + 2)y = x$